

Hierarchical Representation of Plain Areas of Post-Interpolation Residuals for Image Compression

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Abstract—We propose an algorithm for encoding quantized post-interpolation residuals within the framework of hierarchical image compression. This coding algorithm is based on a hierarchical representation of the plain areas of quantized post-interpolation residuals to improve the coding efficiency of these areas. The proposed algorithm reorders the post-interpolation residuals to increase the size of the plain areas. We embed the proposed coding algorithm for post-interpolation residuals into a hierarchical image compression method. This method is based on interpolation the image scale levels using more resampled scale levels of the same image. The errors of this interpolation (post-interpolation residuals) are then quantized and encoded. We use the proposed algorithm to encode the quantized post-interpolation residuals of the hierarchical compression method. We perform computational experiments to study the effectiveness of the proposed algorithm for a set of natural images. We experimentally confirm that the use of the proposed coding algorithm for post-interpolation residuals makes it possible to increase the efficiency of the hierarchical method of image compression.

Keywords—encoding, image interpolation, image compression

I. INTRODUCTION

The size of processed images continues to grow at present, so the problem of studying image compression methods is still relevant. There are a large number of different approaches [1, 2] to image compression. Many of them continue to develop now.

The approach based on discrete orthogonal transformations has become especially widespread [1, 3]. The most famous representative of this approach is the JPEG compression method [4, 5]. The main competitor of this method in terms of efficiency is the JPEG-2000 method [6] based on the wavelet transform [7-9]. Fractal methods [10] of image compression also make it possible to achieve high compression rates.

However, all of these approaches are based on an image transformation into some kind of transformed space. Therefore, these approaches have a common set of disadvantages: high computational complexity of this transformation, significant difficulties of the error control in the transformed space, etc.

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In this article, we consider a hierarchical image compression method [11, 12], which is largely free of the described disadvantages because it does not require a transformation to another space. Among the important advantages of this method, it should be noted that there is also the possibility of hierarchical access to compressed data and wide possibilities for the noise immunity control and for the control of the bit rate of the output compressed data stream.

The considered hierarchical compression method is based on interpolation of samples of resampled image scale levels and subsequent coding of post-interpolation residuals. In this paper, we propose an algorithm for encoding plain areas of the array of quantized post-interpolation residuals. Our algorithm is based on a hierarchical representation of this array. This representation allows us to improve the efficiency of the hierarchical image compression method.

II. HIERARCHICAL IMAGE COMPRESSION

The considered image compression method is based on hierarchical grid interpolation [11, 12]. This method uses a special representation of the image $F = \{f(m, n)\}$ as a set of L disjoint scale levels (see Fig. 1).

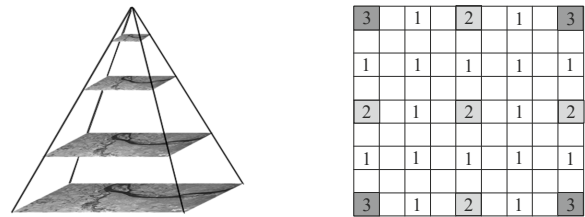


Fig. 1. Hierarchical representation of an image by four scale levels ($L = 4$). The grid contains numbers l of scale levels (empty cells correspond to $l = 0$).

Let's describe this hierarchical image representation in more detail. Let us introduce into consideration the set $I_l^+ = \{(2^l m, 2^l n)\}$ of two-dimensional indices describing the pixels grid of the image resampled with the step of 2^l . One of these grids, corresponding to step 2^{L-1} , defines the so-called «highest» scale level. To get the set I_l of pixel indices for any other scale level, it is necessary to remove pixels with the step of 2^{l+1} from the pixel grid with the step of 2^l :

$$I_{L-1} = I_{L-1}^+, \quad I_l = I_l^+ \setminus I_{l+1}^+, \quad 0 \leq l < L. \quad (1)$$

Therefore, the union of sets of pixel indices of hierarchical levels is a cover (full and non-redundant representation) of the set $I = \{(m, n)\}$ of image pixel indices:

$$I = \bigcup_{l=0}^{L-1} I_l, \quad I_j \cap I_k = \emptyset \quad \forall k \neq j \quad (2)$$

The described hierarchical representation allows us to organize sequential processing of scale image levels during compression: from the «highest» scale level number $(L-1)$ to the lowest scale level number zero.

We do not specify the compression algorithm of the «highest» scale level, since the number of pixels of this level is very small. Compression algorithm for any other level number l consists of the following steps:

1) Interpolation. We calculate the interpolation value $\hat{f}(m, n)$ using the interpolation function P for each pixel $f(m, n)$ of the scale level number l . We use already processed pixels $\bar{f}(m, n)$ of the more resampled scale levels:

$$\hat{f}(m, n) = P\left\{\left\{\bar{f}(j, k), (j, k) \in I_{l+1}^+\right\}\right\}, (m, n) \in I_l. \quad (3)$$

We ensure that the interpolation values are equals during the compression and decompression stages because we use the reconstructed pixel values $\bar{f}(m, n)$, rather than the original pixel values $f(m, n)$, as reference values for interpolation in both of these stages.

2) Calculation of post-interpolation residuals (difference signal):

$$\delta(m, n) = f(m, n) - \hat{f}(m, n), (m, n) \in I_l, \quad (4)$$

3) Quantization of the post-interpolation residuals (we use a uniform scale in this article):

$$\bar{\delta}(m, n) = \left\lfloor \frac{\varepsilon_{\max} + |\delta(m, n)|}{2\varepsilon_{\max} + 1} \right\rfloor \text{sign}(\delta(m, n)), (m, n) \in I_l \quad (5)$$

where $\lfloor \dots \rfloor$ denotes the integer part of the number, and $\text{sign}(\dots)$ denotes the sign of the number:

$$\text{sign}(\delta) = \begin{cases} 1, & \delta \geq 0 \\ -1, & \delta < 0 \end{cases} \quad (6)$$

This quantizer allows us to control the maximum [16] error ε_{\max} between the original $f(m, n)$ and the decompressed $\bar{f}(m, n)$ images:

$$\varepsilon_{\max} = |f(m, n) - \bar{f}(m, n)|. \quad (7)$$

4) Encoding of quantized post-interpolation residuals $\bar{\delta}$ and placing the resulting code sequence in an archive file. Entropy coding algorithms are usually used at this stage.

5) Dequantization and restoration of pixel values (implementation of the decompression within the compression stage):

$$\bar{f}(m, n) = \hat{f}(m, n) + \bar{\delta}(m, n)(2\varepsilon_{\max} + 1), (m, n) \in I_l$$

Therefore, during compression, we calculate the recovered pixel values $\bar{f}(m, n)$ that are equal the values that we will subsequently calculate during the decompression stage. We use these reconstructed values $\bar{f}(m, n)$ to interpolate (3) pixels of the more resampled scale levels.

III. HIERARCHICAL REPRESENTATION OF PLAIN AREAS OF POST-INTERPOLATION RESIDUES

As noted above, entropy encoding algorithms are usually used at the stage of encoding quantized post-interpolation residuals in hierarchical image compression methods. These algorithms can effectively eliminate the statistical redundancy of these residuals. However, these algorithms ignore the two-dimensional nature of these residuals. In other words, the relationships between the encoded samples for one of the dimension is lost when using entropy encoding algorithms.

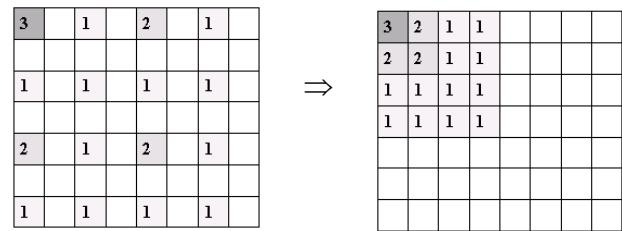


Fig. 2. Reordering of quantized post-interpolation residuals

However, preliminary analysis shows that, it makes sense to consider these relationships for hierarchical compression methods. Indeed, the interpolator works accurately on relatively flat areas of images. This leads to the appearance of plain areas in the array of post-interpolation residuals. Moreover, the size of these areas in the quantized array is even larger. This paper proposes to use a hierarchical algorithm for the efficient coding of these plain areas.

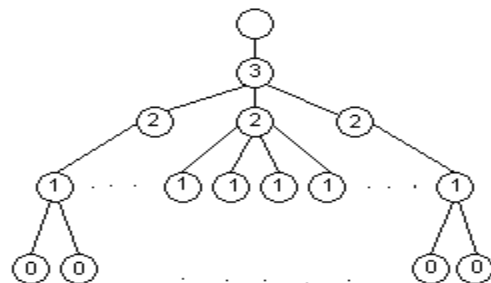


Fig. 3. Hierarchical representation of the array of quantized post-interpolation residuals

We will modify the general hierarchical compression procedure to apply this algorithm as follows. First, we accumulate the values of the quantized post-interpolation

residuals for all scale levels of the image in a single intermediate two-dimensional array. Then we reorder this intermediate array to group the values of post-interpolation residuals located at the same scale levels (see Fig. 2). As a result, the plain areas of the array of quantized post-interpolation residues increase in size.

We represent the reordered intermediate array in the hierarchical structure shown in Fig. 3. In this case, the levels of the resulting hierarchical structure correspond to the scale levels of the original image representation. The presence of a trivial sample in this hierarchical (tree-like) structure with a high probability entails the appearance of the trivial samples at the next levels of the hierarchy. In other words, the described tree of quantized post-interpolation residuals with a high probability contains trivial sub-trees. We discard these trivial sub-trees in subsequent entropy encoding, which leads to a reduction in the compressed data size.

IV. THE EXPERIMENT

To study the efficiency of the proposed algorithm for coding post-interpolation residuals, we have implemented this algorithm in software. We built this algorithm into the hierarchical image compression method. We used the free «Waterloo» image dataset [13] (see Fig. 4) for computational experiments.



Fig. 4. Fragment of the test images dataset "Waterloo"

The quality measure was the relative gain Δ in the archive size (in percent), which was achieved by using the proposed algorithm within the hierarchical compression method:

$$\Delta = \left(\frac{S - S_0}{S} \right) \cdot 100\% , \quad (8)$$

where S_0 and S are archive sizes with and without the proposed algorithm, respectively.

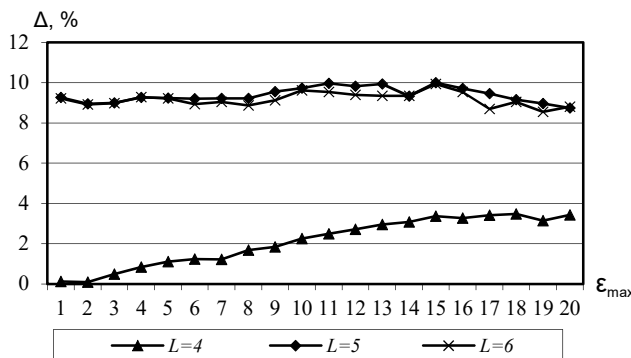


Fig. 5. Dependence of the efficiency Δ of the proposed algorithm of post-interpolation residuals coding on the maximum error ϵ_{max} for the different number L of the image scale levels

The results are in Fig. 5. You can see that the use of the proposed algorithm provides the noticeable (8-10%) gain in efficiency, which allows us to conclude that it is promising for using this algorithm in the applied tasks related to the image compression.

V. CONCLUSION

In this article, we have proposed an algorithm for encoding quantized post-interpolation residuals. This algorithm is based on a hierarchical representation of the plain areas of the array of these residuals. We have considered a hierarchical way of reordering quantized post-interpolation residuals in order to increase the efficiency of coding of these residuals. We have built the proposed algorithm for encoding quantized post-interpolation residuals into a hierarchical image compression method.

We have performed computational experiments to investigate the efficiency of the proposed algorithm in the framework of hierarchical compression. We have experimentally confirmed that the use of the proposed algorithm for encoding plane areas of the quantized post-interpolation residuals array can significantly increase the efficiency of the hierarchical method of image compression.

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