Observer-Based Event-Triggered Approach for Stochastic Networked Control Systems Under Denial of Service Attacks

Ning Zhao, Peng Shi, *Fellow, IEEE*, Wen Xing, *Member, IEEE*, and Jonathon Chambers, *Fellow, IEEE*

***Abstract*—In this article, we investigate the stability analysis and controller synthesis problems for a class of stochastic networked control systems under aperiodic denial-of-service (DoS) jamming attacks. First, an observer is constructed to estimate the unmeasurable states, and then a new adaptive event-triggered mechanism on the basis of the observer is proposed to eliminate the adverse effects of DoS attacks and schedule the transmission instants so as to realize a reduction of transmission burden in the network. Under the proposed event-driven communication scheme, an observer-based controller is designed, and a new switched system with time-varying delays is introduced. Conditions for the underlying systems to be mean-square exponentially stable with a weighted L_2-gain are established. Also, conditions to co-design the observer, the controller, and the event-triggered scheme are developed. A mass-spring-damper mechanical system is used to demonstrate the effectiveness and advantages of the new design techniques.**

***Index Terms*—Denial-of-service (DoS) attacks, mean-square exponential stability, networked control systems (NCSs), weighted L_2-gain.**

I. Introduction

NETWORKED control systems (NCSs) have been extensively studied over the past 20 years and have been widely applied in many fields, such as automotive automation, tactile collaboration on the Internet, transport networks, intelligent buildings, and unmanned aerial vehicles [1]–[8]. In NCSs, the communication network among the sensor, the controller, and the actuator is open and public, and because of its openness, there exist quite a few security vulnerabilities and threats, and the potential for the transmitted data to be manipulated by hostile intruders is enormous. Special attention has been recently raised with respect to network security problems in NCSs, especially from the perspective of control and some instructive results have been achieved [9]–[14].

When it comes to network security, a key issue is to gain a deeper understanding of cyber threat actors. Cyberattacks are classified into denial-of-service (DoS) attacks [9] and deception attacks [14]. The former aims to make a network resource inaccessible to its intended users by continually sending superfluous requests to cause network congestion or paralysis; and the latter attempts to destroy the integrity of the transmitted data by injecting false or misleading information to mask the original data. Compared with deception attacks, DoS attacks need no prior knowledge of the system, but they can seriously affect the timeliness of information exchange and lead to system performance degradation or even instability. Many efforts have been made to design security measures against DoS attacks. For example, in [15]–[17], a probabilistic data packet dropping model was introduced to characterize the impact of DoS attacks and optimal defense strategies were developed to compensate the undesirable effects caused by attacks. Note that this attack modelling approach has its limitations because DoS attacks do not necessarily follow a probability distribution. The motivations behind such malicious acts are relatively rare and can be difficult to prove. In [9], [18], DoS attacks were modeled as a pulsewidth modulated signal due to their energy-constrained nature, detection evasion techniques, and ease of implementation, which are recognized as a form of periodic DoS jamming attacks. In this model, attacks alternate between sleeping and jamming states in a periodic fashion, and the frequency and duration of DoS off/on transitions have to be known in advance. Furthermore, within the framework of aperiodic jamming, DoS attacks with constraints on upper bounds of frequency and duration were investigated, and an impulsive controller with predictor and observer was proposed in [10] to maximize tolerance to DoS attacks. For the same attack model, the work in [11] studied a sampling-logic-based control strategy to ensure the input-to-state stability of the closed-loop NCS. Motivated by these discoveries, in this article, we will explore the challenges posed by aperiodic DoS jamming attacks and propose new countermeasures to prevent the success of DoS attacks in NCSs.

Manuscript received July 27, 2020; accepted October 14, 2020. Date of publication November 4, 2020; date of current version February 26, 2021. This work was partially supported by the National Natural Science Foundation of China under Grant 61773131. Recommended by Associate Editor Sonia Martinez. *(Corresponding author: Wen Xing.)*

Ning Zhao and Wen Xing are with the College of Intelligent Systems Science and Engineering, Harbin Engineering University, Harbin 150001, China (e-mail: zhaoning@hrbeu.edu.cn; xingwen428@126.com).

Peng Shi is with the School of Electrical, and Electronic Engineering, University of Adelaide, Adelaide, SA 5005, Australia (e-mail: peng.shi@adelaide.edu.au).

Jonathon Chambers is with the Department of Engineering, University of Leicester, Leicester LE1 7RH, U.K. (e-mail: Jonathon.Chambers@newcastle.ac.uk).

Digital Object Identifier 10.1109/TCNS.2020.3035760

2325-5870 © 2020 IEEE. Personal use is permitted, but republication/redistribution requires IEEE permission.
See https://www.ieee.org/publications/rights/index.html for more information.

Authorized licensed use limited to: Cornell University Library. Downloaded on May 23,2021 at 20:27:28 UTC from IEEE Xplore. Restrictions apply.

To reduce the communication burden and increase the network resource utilization efficiency, approaches to avoid continuous communication has become an effective and attractive solution. In contrast with traditional time-triggered approaches, event-triggered approaches can adjust the sampling rate according to system dynamics and actual demands and have raised major concerns [19]–[22]. However, only a few studies have taken DoS attacks into consideration. Specifically, the work in [12] developed a periodic event-triggered mechanism to assist in the realization of exponential stability for an NCS subject to DoS attacks. In [18], a periodic event-triggered scheme and a state feedback controller were devised to solve the exponential stability problem for an NCS under uncertainty, quantization, and DoS attacks. It is worth noting that the full information of system states was assumed to be available in the above-mentioned studies. Such an assumption is somewhat conservative in control engineering applications, since system states are usually partially known based on measurable system outputs. Therefore, it is of paramount importance to develop a new event-triggered mechanism and control strategy by means of an appropriate state observer such that the communication burden can be greatly alleviated while preserving the desired performance of an NCS under DoS attacks. To the best of our knowledge, observer-based attack-resilient techniques have not yet been sufficiently explored in literature, which motivates our current work. In addition, due to complex and uncertain internal and external environments, it is more reasonable to consider the NCS as a stochastic system, instead of a deterministic system. Another motivation of this article is to provide stochastic NCSs with safety-oriented and resource-saving countermeasures as well as a systematic performance analysis method.

In this article, we aim to address the problems of mean-square exponential stability and weighted L_2-gain performance analysis for a class of nonlinear stochastic NCSs with unmeasurable states and subject to dual-channel aperiodic DoS jamming attacks. The main idea of this article is to transfer the concerned NCS to a switched system with time-varying delays by developing new state observer, event-triggered communication scheme and control strategy, and establishing a method to integrate the proposed techniques into a unified framework. The main results from this article are as follows:

1) Different from [12], [18] with a periodic attack model and a constant event-triggered threshold, an improved observer-based adaptive event-triggered mechanism is presented in this article to reduce the amount of transmitted data packets as well as the frequency of control updates, while resisting malicious aperiodic attacks so as to help maintain the desired control performance.
2) A secure controller is designed to guarantee that the resultant closed-loop system is not only exponentially stable in the mean-square sense but also fulfills a weighted L_2-gain performance level by resorting to the input delay approach, switched system approach, and stochastic analysis techniques.
3) A solution for jointly designing the observer gain, the control gain, and event-triggered parameters is provided.

Notation: Let $\mathbb{R}^{n\times m}$ be a set of $n\times m$ real matrices. For a symmetric matrix $\mathcal{R}\in\mathbb{R}^{n\times n}$, $\mathcal{R}>0$ ($\mathcal{R}\geq 0$) means that $\mathcal{R}$ is positive definite (semidefinite). The sign $\mathrm{He}(\mathcal{R})$ denotes $\mathcal{R}+\mathcal{R}^T$. The symbol $*$ represents the symmetric term in a symmetric block matrix. Define $||\varphi||_h=\sup_{-h\leq s\leq 0}||\varphi(s)||$, where $||\cdot||$ denotes the Euclidean norm on $\mathbb{R}^n$.

II. Problem Formulation and Preliminaries

A. System Description

Consider a nonlinear stochastic NCS described by

$$\begin{cases}\mathrm{d}x(t)=[Ax(t)+Bu(t)+B_w w(t)]\mathrm{d}t+D\psi(x(t))\mathrm{d}\varpi(t),\\ y(t)=Cx(t)+Ev(t)\end{cases}\tag{1}$$

where $x(t)\in\mathbb{R}^n$, $u(t)\in\mathbb{R}^m$, $w(t)\in\mathbb{R}^p$, $y(t)\in\mathbb{R}^q$, and $v(t)\in\mathbb{R}^v$ are state vector, control input, external disturbance, measurement output, and measurement noise, respectively. The diffusion scaling $\psi(\cdot):\mathbb{R}^n\rightarrow\mathbb{R}$ is a nonlinear function which satisfies $||\psi(x(t))||^2\leq\chi||x(t)||^2$ for all $x\in\mathbb{R}^n$ with $\chi>0$ being a known constant. $\varpi(t)$ is a 1-D Brownian motion having properties of $\mathbf{E}\{\mathrm{d}\varpi(t)\}=0$ and $\mathbf{E}\{\mathrm{d}\varpi^2(t)\}=\mathrm{d}t$. A, B, B_w, C, D and E are known matrices with appropriate dimensions. Here, (i) state vector $x(t)$ is unmeasurable; (ii) $\mathrm{rank}(B)=m$; and (iii) pairs (A,B) and (C,A) are controllable and observable, respectively.

B. Aperiodic DoS Jamming Attacks

Inspired by [9], dual-channel aperiodic DoS jamming attacks are considered in this section. The attacks intend to block communication channels intermittently and their damage exists in both sensor-to-controller and controller-to-actuator channels. The attack model is as follows:

$$\mathcal{W}_{DoS}(t)=\begin{cases}0,\ t\in[d_n,d_n+s_n),\\ 1,\ t\in[d_n+s_n,d_{n+1})\end{cases}\tag{2}$$

where time sequences $\{d_n\}_{n\in\mathbb{N}}$ and $\{s_n\}_{n\in\mathbb{N}}$ satisfy $0=d_0<d_0+s_0\leq d_1<\cdots<d_{n-1}<d_{n-1}+s_{n-1}\leq d_n<\cdots$ for $n\in\mathbb{N}$. During the nth time interval, $[d_n,d_n+s_n)$ represents the sleeping period over which attacks are not in action, and communication is possible, while $[d_n+s_n,d_{n+1})$ denotes the jamming period where attacks are active and communication is denied.

For any $t\geq t_*\geq 0$, let $n(t,t_*)$ represent the total number of DoS OFF/ON transitions occurring over interval $[t_*,t)$, and let $\Pi(t,t_*)$ denote the sets of attack duration over interval $[t_*,t)$. Similar to [11], some limitations are imposed on both DoS frequency and duration by adding the following assumptions.

Assumption 1 (DoS frequency): For any $t\geq t_*\geq 0$, there exist $v_1\geq 0$ and $T_{D1}>0$ such that

$$n(t,t_*)\leq v_1+\frac{t-t_*}{T_{D1}}.$$

Authorized licensed use limited to: Cornell University Library. Downloaded on May 23,2021 at 20:27:28 UTC from IEEE Xplore. Restrictions apply.

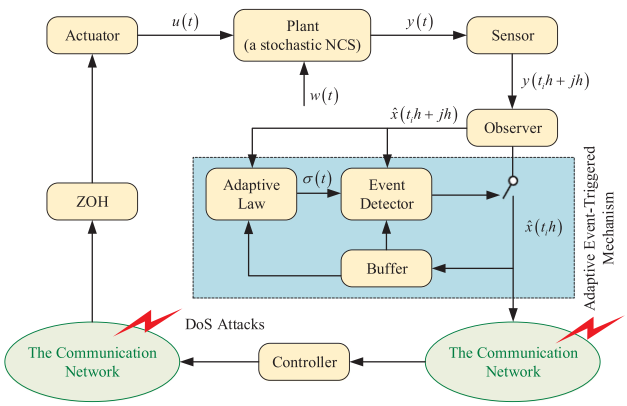


Figure 1. Framework of a stochastic NCS under DoS attacks.

Assumption 2 (DoS duration): For any $t \geq t_* \geq 0$, there exist $v_2 \geq 0$ and $T_{D2} > 1$ such that

$$|\Pi(t, t_*)| \leq v_2 + \frac{t - t_*}{T_{D2}}.$$

C. An Observer-Based Adaptive Event-Triggered Scheme Against DoS Attacks

In this section, we will design an observer to estimate system states, since states of system (1) are imperfectly measurable. Based on the observer, an adaptive event-triggered mechanism that is resilient against DoS attacks will be implemented. For clarity, we first introduce the operation process of an NCS under DoS attacks. As shown in Fig. 1, the measurable output $y(t)$ is periodically sampled by the sampler with a sampling period $h > 0$. Then, the sampled data $y(kh)$, $k \in \mathbb{N}$, is instantly sent to the observer which is in charge of generating the estimated state $\hat{x}(kh)$. At each sampling time, an event-triggered mechanism is adopted to determine the releasing instant of the estimated state. Whether state $\hat{x}(kh)$ is allowed to be released depends on a predefined event-triggered condition embedded in the event detector. If the event-triggered condition is not satisfied at time instant $t_i h$ $(i \in \mathbb{N}, t_i \in \mathbb{N})$, then the estimated state $\hat{x}(t_i h)$ is authorized to be transmitted through the communication network. Let $\{t_0 h(= 0), t_1 h, t_2 h, \ldots\} \subset \mathbb{N}$ be the releasing time sequence, where $t_i h$ and $t_{i+1} h$ denote current and next releasing time instants, respectively.

For all $t \in [t_i h, t_{i+1} h)$, the observer is given as

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(t_i h + jh) - C\hat{x}(t_i h + jh)) \tag{3}$$

where $j \in \{j \in \mathbb{N} | t_i h + jh \leq t_{i+1} h\}$, $\hat{x}(t) \in \mathbb{R}^n$ is the estimated state vector, and $L \in \mathbb{R}^{n \times q}$ is the observer gain matrix to be determined later.

In order to save communication resources, an observer–based adaptive event-triggered scheme is proposed to determine when the newly estimated state will be sent out to the controller. The corresponding event-triggered condition is defined as

$$t_{i+1} h = t_i h + \min_{j \in \mathbb{N}, j \geq 1} \{jh | e^T(t_i h + jh) \Psi e(t_i h + jh) > \sigma(t_i h + jh) \hat{x}^T(t_i h) \Psi \hat{x}(t_i h)\} \tag{4}$$

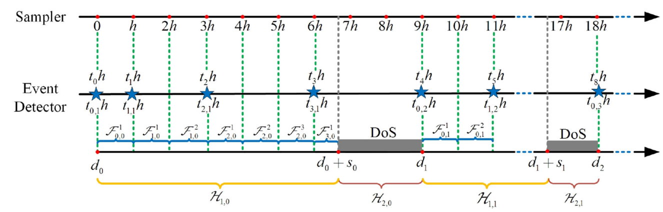


Figure 2. Transmission instants under aperiodic DoS attacks. $\star$: Transmission instants.

where $e(t_i h + jh) = \hat{x}(t_i h) - \hat{x}(t_i h + jh)$, $\Psi \in \mathbb{R}^{n \times n} > 0$ is a weighting matrix to be designed, and threshold $\sigma(t_i h + jh) \in [\sigma_0, \sigma_1]$ is a dynamic variable which changes according to the following adaptive law:

$$\sigma(t_i h + jh) = \sigma_0 + (\sigma_1 - \sigma_0)\, \mathrm{e}^{-\kappa_1 \| \|\hat{x}(t_i h)\| - \|\hat{x}(t_i h + jh)\| \|^{\kappa_2}} \tag{5}$$

where $\kappa_1 > 0$ and $\kappa_2 \geq 1$ are two constants being used to adjust the sensitivity of the threshold and $0 \leq \sigma_0 \leq \sigma_1 < 1$.

Note that due to the aperiodic occurrence of DoS attacks (2), data transmission via the communication network is interrupted at different intervals. This makes event-triggered condition (4) not applicable to the stability problem of the system (1). To tackle the problem, a new attack-resilient event-triggered condition is developed based on adaptive event-triggered condition (4), which is presented as follows:

$$t_{i,n+1} h \in \{t_r h \text{ satisfying } (4) | t_r h \in \mathcal{H}_{1,n}, t_r \in \mathbb{N}\} \cup \{d_n\} \tag{6}$$

where $\mathcal{H}_{1,n} := [d_n, d_n + s_n)$, and $t_{i,n+1} h$ represents the time instant when the estimated state is successfully transmitted to the controller for $i \in \mathcal{I}(n) = \{0, 1, \ldots, i(n)\}$ and $i(n) = \max\{i \in \mathbb{N} | d_n + s_n > t_{i,n+1} h\}$.

An example of event-triggered time instants for data transmission under DoS attacks (2) is depicted in Fig. 2.

Remark 1: In this article, the event-triggered scheme (4) is presented for attack-free scenarios. Compared with traditional event-triggered schemes where dedicated hardware for continuous state monitoring is required and the threshold parameter is always defined as a constant value. The event-triggered scheme developed in this article is established based on the sampled data as well as a dynamic threshold parameter. As such, the minimum time interval between two consecutive transmission instants is no less than a sampling period. The Zeno phenomenon can be absolutely ruled out, without additional theoretical analysis and calculation. For the improvement and expansion of scheme (4) related to attack scenarios, the resilient event-triggered scheme (6) is proposed and employed as a security countermeasure to prevent DoS attacks. The scheme (6) allows the sampled data to be released immediately when attacks switch from active mode to sleep mode. It can realize compensation to the attack inflicted performance penalties by appropriately increasing the amount of the released data.

Remark 2: In contrast to the related works [12], [18], a dynamic threshold introduced in (5) is incorporated into

Authorized licensed use limited to: Cornell University Library. Downloaded on May 23,2021 at 20:27:28 UTC from IEEE Xplore. Restrictions apply.

attack-resilient event-triggered scheme (6), which not only counteracts serious dual-channel DoS attacks, but also overcome the bad effects caused by the error between the latest releasing data and the current sampled data with a small cost. More specifically, in adaptive rule (5), the threshold $\sigma(t_ih+jh)$ is dynamically adjusted according to $|||\hat{x}(t_ih)||-||\hat{x}(t_ih+jh)|||$ and two adjustable parameters κ_1 and κ_2, which indicates that adaptive rule (5) provides more flexibility for the threshold in changing with the evolution of the system. As the value of $|||\hat{x}(t_ih)||-||\hat{x}(t_ih+jh)|||$ gets bigger, the threshold gets smaller. Then, scheme (6) uses a smaller threshold value to speed up the frequency of triggers, ensuring that the controller can be updated frequently enough to achieve a better system performance. On the contrary, the threshold becomes bigger when the value of $|||\hat{x}(t_ih)||-||\hat{x}(t_ih+jh)|||$ becomes smaller. The scheme (6) uses a larger threshold value to obtain a lower frequency of event triggers, which can save more communication bandwidth.

D. Secure Controller Design and Switched System Formulation

When dual-channel DoS attacks occur, the data packet released by the event-triggered scheme is lost and unable to arrive at the controller. The control signal generated by the controller is intercepted and cannot reach the actuator successfully as well. In this case, it is quite essential to develop a secure control strategy for the considered NCS (1) to achieve the desired closed-loop performance even though attacks exist in the network communication. The control input $u(t)$ based on the observer state is

$$u(t)=\begin{cases}K\hat{x}(t_{i,n+1}h),\ t\in[t_{i,n+1}h,t_{i+1,n+1}h)\cap\mathcal{H}_{1,n},\\ 0_{m\times 1},\ t\in\mathcal{H}_{2,n}\end{cases}\tag{7}$$

where $\mathcal{H}_{2,n}:=[d_n+s_n,d_{n+1})$, and $K\in\mathbb{R}^{m\times n}$ is the control gain matrix to be designed later.

Under control strategy (7), in order to analyze the system stability by virtue of the input delay method and the switched system approach, our primary aim is to convert NCS (1) under attacks (2) and event-triggered communication mechanism (6) to a switched system with time delays. For this purpose, we divide interval $\mathcal{H}_{1,n}$ into the following subintervals:

$$\mathcal{H}_{1,n}=\cup_{i=0}^{i(n)}\cup_{k=1}^{\eta_{i,n}+1}\mathcal{F}_{i,n}^k$$

where

$$\begin{cases}\mathcal{F}_{i,n}^k=[t_{i,n+1}h+(k-1)h,t_{i,n+1}h+kh),\\ \qquad k=1,2,\ldots,\eta_{i,n},\\ \mathcal{F}_{i,n}^{\eta_{i,n}+1}=[t_{i,n+1}h+\eta_{i,n}h,t_{i+1,n+1}h),\\ \mathcal{F}_{i(n),n}^k=[t_{i(n),n+1}h+(k-1)h,t_{i(n),n+1}h+kh),\\ \qquad k=1,2,\ldots,\eta_{i(n),n},\\ \mathcal{F}_{i(n),n}^{\eta_{i(n),n}+1}=[t_{i(n),n+1}h+\eta_{i(n),n}h,d_n+s_n)\end{cases}\tag{8}$$

in which $\eta_{i,n}\triangleq\max\{k\in\mathbb{N}|t_{i,n+1}h+kh<t_{i+1,n+1}h\}$ for $i\in\mathcal{I}_n-\{i(n)\}$ and $\eta_{i(n),n}\triangleq\max\{k\in\mathbb{N}|t_{i(n),n+1}h+kh<d_n+s_n\}$.

Now, we define two piecewise functions, which are input delay $\tau_{i,n}(t)$ and measurement error $e_{i,n}(t)$. The corresponding definitions are as follows:

$$\tau_{i,n}(t)=\begin{cases}t-t_{i,n+1}h,t\in[t_{i,n+1}h,t_{i,n+1}h+h),\\ t-t_{i,n+1}h-h,t\in[t_{i,n+1}h+h,t_{i,n+1}h+2\,h),\\ \vdots\\ t-t_{i,n+1}h-\tilde{\eta}_{i,n}h,t\in[t_{i,n+1}h+\tilde{\eta}_{i,n}h,\\ \qquad t_{i,n+1}h+(\tilde{\eta}_{i,n}+1)h),\end{cases}$$

where $\tilde{\eta}_{i,n}=\eta_{i,n}$ for $i\in\mathcal{I}_n-\{i(n)\}$ and $\tilde{\eta}_{i(n),n}\triangleq\max\{k\in\mathbb{N}|t_{i(n),n+1}h+kh<d_{n+1}\}$.

$$e_{i,n}(t)=\begin{cases}0,t\in\mathcal{F}_{i,n}^1,\\ \hat{x}(t_{i,n+1}h)-\hat{x}(t_{i,n+1}h+h),t\in\mathcal{F}_{i,n}^2,\\ \vdots\\ \hat{x}(t_{i,n+1}h)-\hat{x}(t_{i,n+1}h+\eta_{i,n}h),t\in\mathcal{F}_{i,n}^{\eta_{i,n}+1}.\end{cases}$$

According to the definitions of $\tau_{i,n}(t)$ and $e_{i,n}(t)$, the successfully transmitted data $\hat{x}(t_{i,n+1}h)$ is rewritten as, for $t\in\mathcal{F}_{i,n}^k\cap\mathcal{H}_{1,n}$

$$\hat{x}(t_{i,n+1}h)=e_{i,n}(t)+\hat{x}(t-\tau_{i,n}(t)).\tag{9}$$

Let $\tilde{x}(t)=x(t)-\hat{x}(t)$ be the estimation error. By integrating (1) with (3), the following error system is reformulated:

$$\begin{aligned}\mathrm{d}\tilde{x}(t)=&[A\tilde{x}(t)-LC\tilde{x}(t-\tau_{i,n}(t))+B_ww(t)\\ &-LEv(t-\tau_{i,n}(t))]\mathrm{d}t+D\psi(x(t))\mathrm{d}\varpi(t).\end{aligned}\tag{10}$$

Based on (1), (7), (9), and (10), by denoting $\xi(t)=[x^T(t)\ \ \tilde{x}^T(t)]^T$ and $\bar{w}(t)=[w^T(t)\ \ v^T(t-\tau_{i,n}(t))]^T$, an augmented switched system with time delays is created as follows:

$$\begin{cases}\mathrm{d}\xi(t)=\begin{cases}[A_1\xi(t)+\bar{A}_1\xi(t-\tau_{i,n}(t))+E_1e_{i,n}(t)\\ +\bar{B}_w\bar{w}(t)]\mathrm{d}t+\bar{D}\psi(x(t))\mathrm{d}\varpi,\ t\in\mathcal{F}_{i,n}^k\cap\mathcal{H}_{1,n},\\ [A_2\xi(t)+\bar{A}_2\xi(t-\tau_{i,n}(t))+\bar{B}_w\bar{w}(t)]\mathrm{d}t\\ +\bar{D}\psi(x(t))\mathrm{d}\varpi,\ t\in\mathcal{H}_{2,n},\end{cases}\\ \xi(\theta)=\varphi(\theta),\ \theta\in[-h,0]\end{cases}\tag{11}$$

where

$$A_1=A_2=\begin{bmatrix}A&0_{n\times n}\\0_{n\times n}&A\end{bmatrix},\ \bar{A}_1=\begin{bmatrix}BK&-BK\\0_{n\times n}&-LC\end{bmatrix},$$

$$\bar{A}_2=\begin{bmatrix}0_{n\times n}&0_{n\times n}\\0_{n\times n}&-LC\end{bmatrix},\ E_1=\begin{bmatrix}BK\\0_{n\times n}\end{bmatrix},$$

$$\bar{B}_w=\begin{bmatrix}B_w&0_{n\times v}\\B_w&LE\end{bmatrix},\ \bar{D}=\begin{bmatrix}D\\D\end{bmatrix}.$$

Before proceeding, the main purpose to be achieved in this article is summarized below.

The objective of this article is to design observer (3), attack-resilient event generator (6) and security-oriented controller (7) such that the system described by (11) meets the following two requirements:

1) The closed-loop system (11) with $\bar{w}(t)=0_{(p+v)\times 1}$ is exponentially stable in the mean-square sense, which

Authorized licensed use limited to: Cornell University Library. Downloaded on May 23,2021 at 20:27:28 UTC from IEEE Xplore. Restrictions apply.

implies that for any initial condition, there exist constants $\nu \geq 1$ and $\upsilon > 0$ such that

$$\mathbf{E}\{||\xi(t)||^2\} \leq \nu e^{-\upsilon t}||\varphi||_h^2,\ t \geq 0. \tag{12}$$

2) Under zero initial condition, for given $\alpha > 0$ and $\tilde{\gamma} > 0$, the measurement output satisfies

$$\int_0^\infty e^{-\alpha s}\mathbf{E}\{y^T(s)y(s)\}\mathrm{d}s \leq \tilde{\gamma}^2 \int_0^\infty \bar{w}^T(s)\bar{w}(s)\mathrm{d}s \tag{13}$$

for any nonzero $\bar{w}(t)$.

III. Main Results

We now state and establish the first result in this article.

Theorem 1: Let $\gamma > 0$, $\chi > 0$, $\rho_j > 0$ $(j = 1, 2)$, and matrices L and K be given. For scalars $\sigma_1 \in [0, 1)$, $T_{D1} > 0$, $T_{D2} > 1$, $\alpha_j > 0$, $\mu_j > 1$ $(j = 1, 2)$ and $h > 0$ satisfying

$$\lambda := 2\alpha_1 - \frac{2(\alpha_1+\alpha_2)h + \ln(\mu_1\mu_2)}{T_{D1}} - \frac{2(\alpha_1+\alpha_2)}{T_{D2}} > 0 \tag{14}$$

if there exist matrices $\Psi > 0$, $P_j > 0$, $Q_j > 0$, $R_j > 0$, $Z_j > 0$, S_j, M_j and N_j $(j = 1, 2)$ with appropriate dimensions such that for $j = 1, 2$

$$P_1 \leq \mu_2 P_2, P_2 \leq \mu_1 e^{2(\alpha_1+\alpha_2)h} P_1, \tag{15}$$

$$\zeta_j \leq \mu_{3-j}\zeta_{3-j}, \zeta \in \{Q, R, Z\}, \tag{16}$$

$$\bar{D}^T P_j \bar{D} - \rho_j I \leq 0, \tag{17}$$

$$\Xi_j = \begin{bmatrix} \Xi_{j11} & \Xi_{j12} \\ * & \Xi_{j22} \end{bmatrix} < 0 \tag{18}$$

where

$$\Xi_{j11} = \Upsilon_{j1} + \Upsilon_{j2} + \Upsilon_{j2}^T,\ \Upsilon_{j1} = \begin{bmatrix} \Upsilon_{j11} & \Upsilon_{j12} \\ * & \Upsilon_{j22} \end{bmatrix},$$

$$\Upsilon_{j11} = \mathrm{He}\{P_j A_j\} + \rho_j\chi I_{2n} + Q_j - 2(-1)^j\alpha_j P_j,$$

$$\Upsilon_{112} = \begin{bmatrix} P_1\bar{A}_1 & 0_{2n\times 2n} & P_1 E_1 & P_1\bar{B}_w \end{bmatrix},$$

$$\Upsilon_{212} = \begin{bmatrix} P_2\bar{A}_2 & 0_{2n\times 2n} & P_2\bar{B}_w \end{bmatrix},$$

$$\Upsilon_{122} = \mathrm{diag}\{\sigma_1\Gamma^T\Psi\Gamma, -e^{-2\alpha_1 h}Q_1, (\sigma_1 - 1)\Psi, -\gamma^2 I_{p+v}\}$$

$$\Gamma = \begin{bmatrix} I_n & -I_n \end{bmatrix},\ I_1 = \begin{bmatrix} I_{2n} & 0_{2n\times(3n+p+v)} \end{bmatrix}^T,$$

$$I_2 = \begin{bmatrix} 0_{n\times 4n} & I_n & 0_{n\times(p+v)} \end{bmatrix},$$

$$\Upsilon_{222} = \mathrm{diag}\{0_{2n\times 2n}, -e^{2\alpha_2 h}Q_2, -\gamma^2 I_{p+v}\},$$

$$\Upsilon_{j2} = \begin{bmatrix} S_j + M_j & -M_j + N_j & -S_j - N_j & \bar{O}_j \end{bmatrix},$$

$$\bar{O}_j = 0_{((8-j)n+p+v)\times((2-j)n+p+v)},$$

$$\Xi_{j12} = \begin{bmatrix} h^{\frac{1}{2}}S_j & h^{\frac{1}{2}}M_j & h^{\frac{1}{2}}N_j & h^{\frac{1}{2}}X_j^T P_j^T & G_j \end{bmatrix},$$

$$X_1 = \begin{bmatrix} A_1 & \bar{A}_1 & 0_{2n\times 2n} & E_1 & \bar{B}_w \end{bmatrix},$$

$$X_2 = \begin{bmatrix} A_2 & \bar{A}_2 & 0_{2n\times 2n} & \bar{B}_w \end{bmatrix},$$

$$G_j = \begin{bmatrix} \bar{C} & 0_{q\times(6-j)n} & \bar{E} \end{bmatrix}^T,$$

$$\bar{C} = \begin{bmatrix} C & 0_{q\times n} \end{bmatrix},\ \bar{E} = \begin{bmatrix} 0_{q\times p} & E \end{bmatrix},$$

$$\Xi_{j22} = \mathrm{diag}\{\Xi_{j2}, \Xi_{j3}, \Xi_{j4}, \Xi_{j5}, \Xi_{j6}\},$$

$$\Xi_{j2} = -e^{(-1)^j(4-2j)\alpha_j h}R_j,$$

$$\Xi_{j3} = \Xi_{j4} = -e^{(-1)^j(4-2j)\alpha_j h}Z_j,$$

$$\Xi_{j5} = -P_j(R_j + Z_j)^{-1}P_j,\ \Xi_{j6} = -I_q$$

then system (11) is mean-square exponentially stable with a L_2-gain level $\tilde{\gamma} = \gamma\sqrt{2\alpha_1 e^c\mu_2/\lambda}$, where $c = (2(\alpha_1+\alpha_2)h + \ln(\mu_1\mu_2))v_1 + 2(\alpha_1+\alpha_2)v_2$.

Proof: Construct a Lyapunov–Krasovskii functional candidate for system (11) as

$$V_j(t) = \sum_{k=1}^{4} V_j^k(t) \tag{19}$$

where

$$V_j^1(t) = \xi^T(t)P_j\xi(t),$$

$$V_j^2(t) = \int_{t-h}^{t} g(t,s)\xi^T(s)Q_j\xi(s)\mathrm{d}s,$$

$$V_j^3(t) = \int_{-h}^{0}\int_{t+v}^{t} g(t,s)f_j^T(s)R_j f_j(s)\mathrm{d}s\mathrm{d}v,$$

$$V_j^4(t) = \int_{-h}^{0}\int_{t+v}^{t} g(t,s)f_j^T(s)Z_j f_j(s)\mathrm{d}s\mathrm{d}v,$$

$$f_j(t) := A_j\xi(t) + \bar{A}_j\xi(t - \tau_{i,n}(t)) + E_j e_{i,n}(t) + \bar{B}_w\bar{w}(t),$$

$$g(t,s) := e^{(-1)^j 2\alpha_j(t-s)}.$$

When $j = 1$, by using the weak infinitesimal operator $\mathcal{L}$ and the Leibniz–Newton formula of the integration form, we can get that

$$\mathbf{E}\{\mathcal{L}V_1^1(t)\} = \mathbf{E}\{-2\alpha_1 V_1^1(t) + 2\alpha_1\xi_1^T(t)P_1\xi_1(t) + 2\xi^T(t)P_1 f(t) + \psi^T(x(t))\bar{D}^T P_1\bar{D}\psi(x(t))\}, \tag{20a}$$

$$\mathbf{E}\{\mathcal{L}V_1^2(t)\} = \mathbf{E}\{-2\alpha_1 V_1^2(t) + \xi^T(t)Q_1\xi(t) - e^{-2\alpha_1 h}\xi^T(t-h)Q_1\xi(t-h)\}, \tag{20b}$$

$$\mathbf{E}\{\mathcal{L}V_1^3(t)\} \leq \mathbf{E}\Big\{-2\alpha_1 V_1^3(t) + hf^T(t)R_1 f(t) - e^{-2\alpha_1 h}\int_{t-h}^{t} f^T(s)R_1 f(s)\mathrm{d}s\Big\} + \mathbf{E}\Big\{2\eta^T(t)S_1\Big(\xi(t) - \xi(t-h) - \int_{t-h}^{t} f(s)\mathrm{d}s\Big)\Big\}, \tag{20c}$$

$$\mathbf{E}\{\mathcal{L}V_1^4(t)\} \leq \mathbf{E}\Big\{-2\alpha_1 V_1^4(t) + hf^T(t)Z_1 f(t)$$

Authorized licensed use limited to: Cornell University Library. Downloaded on May 23,2021 at 20:27:28 UTC from IEEE Xplore. Restrictions apply.

$$
\begin{aligned}
&- e^{-2\alpha_1 h}\int_{t-h}^{t} f^T(s)Z_1 f(s)\mathrm{d}s\Big\}\\
&+\mathbf{E}\Big\{2\eta^T(t)M_1\left(\xi(t)-\xi(t-\tau_{i,n}(t))\right.\\
&\left.-\int_{t-\tau_{i,n}(t)}^{t} f(s)\mathrm{d}s\right)\Big\}\\
&+\mathbf{E}\Big\{2\eta^T(t)N_1\left(\xi(t-\tau_{i,n}(t))-\xi(t-h)\right.\\
&\left.-\int_{t-h}^{t-\tau_{i,n}(t)} f(s)\mathrm{d}s\right)\Big\}
\end{aligned}\tag{20d}
$$

where $\eta(t)=[\xi^T(t)\ \xi^T(t-\tau_{i,n}(t))\ \xi^T(t-h)\ e_{i,n}^T(t)w^T(t)]^T$. By applying Young's inequality and Jenson's inequality to deal with integral terms in (20c) and (20d), we have

$$
\begin{aligned}
&-2\mathbf{E}\Big\{\eta_1(t)S_1\int_{t-h}^{t} f_1(s)\mathrm{d}s\}\\
&\le he^{2\alpha_1 h}\mathbf{E}\{\eta_1(t)S_1R_1{}^{-1}S_1^T\eta_1(t)\}\\
&\quad+e^{-2\alpha_1 h}\mathbf{E}\Big\{\int_{t-h}^{t} f_1^T(s)R_1f_1(s)\mathrm{d}s\Big\},\\
&-2\mathbf{E}\{\eta_1(t)M_1\int_{t-\tau_{i,n}(t)}^{t} f_1(s)\mathrm{d}s\}\\
&\le he^{2\alpha_1 h}\mathbf{E}\{\eta_1(t)M_1Z_1{}^{-1}M_1^T\eta_1(t)\}\\
&\quad+e^{-2\alpha_1 h}\mathbf{E}\Big\{\int_{t-\tau_{i,n}(t)}^{t} f_1^T(s)Z_1f_1(s)\mathrm{d}s\Big\},\\
&-2\mathbf{E}\{\eta_1(t)N_1\int_{t-h}^{t-\tau_{i,n}(t)} f_1(s)\mathrm{d}s\}\\
&\le he^{2\alpha_1 h}\mathbf{E}\{\eta_1(t)N_1Z_1{}^{-1}N_1^T\eta_1(t)\}\\
&\quad+e^{-2\alpha_1 h}\mathbf{E}\Big\{\int_{t-h}^{t-\tau_{i,n}(t)} f_1^T(s)Z_1f_1(s)\mathrm{d}s\Big\}.
\end{aligned}\tag{21}
$$

From (17), one can obtain

$$\psi^T(x(t))\bar{D}^TP_1\bar{D}\psi(x(t))\le \rho_1\chi x^T(t)x(t).\tag{22}$$

From (4) and (9), one can see that

$$
\begin{aligned}
e_{i,n}^T(t)\Psi e_{i,n}(t)&\le \sigma_1(e_{i,n}(t)+\hat{x}(t-\tau_{i,n}(t)))^T\Psi\\
&\quad\times(e_{i,n}(t)+\hat{x}(t-\tau_{i,n}(t))).
\end{aligned}\tag{23}
$$

Then, for analyzing the weighted L_2-gain performance of system (11), we construct $\Theta(t)=y^T(t)y(t)-\gamma^2\bar{w}^T(t)\bar{w}(t)$. Combining (20a)–(23) yields

$$
\begin{aligned}
&\mathbf{E}\{\mathcal{L}V_1(t)+\Theta(t)\}\\
&\le \mathbf{E}\{-2\alpha_1V_1(t)[\Xi_{111}+he^{2\alpha_1 h}(S_1R_1^{-1}S_1^T+M_1Z_1^{-1}M_1^T\\
&\quad+N_1Z_1^{-1}N_1^T)+G_1G_1^T+hX_1^T(R_1+Z_1)X_1]\eta_1(t)\}.
\end{aligned}
$$

Taking the Schur complement of the matrix $\Xi_1<0$ in (18), we obtain

$$
\begin{aligned}
&\Xi_{111}+he^{2\alpha_1 h}(S_1R_1^{-1}S_1^T+M_1Z_1^{-1}M_1^T+N_1Z_1^{-1}N_1^T)\\
&\quad+G_1G_1^T+hX_1^T(R_1+Z_1)X_1<0
\end{aligned}
$$

which implies

$$\mathbf{E}\{\mathcal{L}V_1(t)+2\alpha_1V_1(t)+\Theta(t)\}\le 0.\tag{24}$$

Using the same analysis method as mentioned in case $j=1$, along the trajectory of system (11) with $j=2$, we obtain

$$
\begin{aligned}
&\mathbf{E}\{\mathcal{L}V_2(t)+\Theta(t)\}\\
&\le \mathbf{E}\{2\alpha_2V_2(t)+\eta_2(t)[\Xi_{211}+h(S_2R_2^{-1}S_2^T+M_2Z_2^{-1}M_2^T\\
&\quad+N_2Z_2^{-1}N_2^T)+G_2G_2^T+hX_2^T(R_2+Z_2)X_2]\eta_2(t)\}
\end{aligned}
$$

where $\eta_2(t)=[\xi^T(t)\ \ \xi^T(t-\tau_{i,n}(t))\ \ \xi^T(t-h)\ \ \bar{w}^T(t)]^T$. Invoking $\Xi_2<0$ in (18), we deduce that

$$\mathbf{E}\{\mathcal{L}V_2(t)-2\alpha_2V_2(t)+\Theta(t)\}\le 0.\tag{25}$$

Furthermore, we choose a piecewise Lyapunov functional as follows:

$$V(t)=\begin{cases}V_1(t),\ t\in\mathcal{H}_{1,n},\\ V_2(t),\ t\in\mathcal{H}_{2,n}.\end{cases}$$

By (24) and (25) with $\bar{w}(t)=0_{(q+v)\times 1}$, there holds

$$\mathbf{E}\{V(t)\}\le\begin{cases}e^{-2\alpha_1(t-d_n)}\mathbf{E}\{V_1(d_n)\},\ t\in\mathcal{H}_{1,n},\\ e^{2\alpha_2(t-d_n-s_n)}\mathbf{E}\{V_2(d_n+s_n)\},\ t\in\mathcal{H}_{2,n}.\end{cases}\tag{26}$$

Let $\varepsilon=e^{\delta h}$ with $\delta=2(\alpha_1+\alpha_2)$. In what follows, we first analyze the weighted L_2-gain performance of system (11). For any nonzero $\bar{w}(t)$, two cases are considered as follows:

Case 1: If $t\in\mathcal{H}_{1,n}$, it follows from (15), (16), and (24) that

$$
\begin{aligned}
&\mathbf{E}\{V(t)\}\\
&\le e^{-2\alpha_1(t-d_n)}\mathbf{E}\{V_1(d_n)\}-\int_{d_n}^{t} e^{-2\alpha_1(t-s)}\mathbf{E}\{\Theta(s)\}\mathrm{d}s\\
&\le \mu_2e^{-2\alpha_1(t-d_n)}\mathbf{E}\{V_2(d_n^-)\}-\int_{d_n}^{t} e^{-2\alpha_1(t-s)}\mathbf{E}\{\Theta(s)\}\mathrm{d}s\\
&\le \mu_2e^{-2\alpha_1(t-(d_{n-1}+s_{n-1}))+\delta(d_n-(d_{n-1}+s_{n-1}))}\\
&\times\mathbf{E}\{V_2(d_{n-1}+s_{n-1})\}\\
&-\mu_2\int_{d_{n-1}+s_{n-1}}^{d_n} e^{-2\alpha_1(t-s)+\delta(d_n-s)}\mathbf{E}\{\Theta(s)\}\mathrm{d}s\\
&-\int_{d_n}^{t} e^{-2\alpha_1(t-s)}\mathbf{E}\{\Theta(s)\}\mathrm{d}s\\
&\ \vdots\\
&\le e^{-2\alpha_1 t+r(t,0)}V_1(0)-\mathcal{V}_1(t)
\end{aligned}\tag{27}
$$

Authorized licensed use limited to: Cornell University Library. Downloaded on May 23,2021 at 20:27:28 UTC from IEEE Xplore. Restrictions apply.

where

$$\mathcal{V}_1(t) = \mu_2 \sum_{k=1}^{n} \int_{d_{k-1}+s_{k-1}}^{d_k} e^{-2\alpha_1(t-s)+r(t,s)} \mathbf{E}\{\Theta(s)\}\mathrm{d}s + \sum_{k=0}^{n-1} \int_{d_k}^{d_k+s_k} e^{-2\alpha_1(t-s)+r(t,s)} \mathbf{E}\{\Theta(s)\}\mathrm{d}s + \int_{d_n}^{t} e^{-2\alpha_1(t-s)} \mathbf{E}\{\Theta(s)\}\mathrm{d}s \quad (28)$$

and $r(t,s) = n(t,s)\ln(\varepsilon\mu_1\mu_2) + \delta|\Pi(t,s)|$. Under zero initial condition, from (27), we have $\mathcal{V}_1(t) \le 0$. By virtue of Assumptions 1 and 2, we obtain

$$\mathcal{V}_{1,\bar{w}}(t) \le \mu_2 e^c \int_0^t e^{-\lambda(t-s)} \bar{w}^T(s)\bar{w}(s)\mathrm{d}s \quad (29)$$

where $\mathcal{V}_{1,\bar{w}}(t)$ is obtained from $\mathcal{V}_1(t)$ by replacing $\Theta(s)$ with $\bar{w}^T(s)\bar{w}(s)$. On the other hand, we know

$$\mathcal{V}_{1,y}(t) \ge \int_0^t e^{-2\alpha_1 t} y^T(s)y(s)\mathrm{d}s \quad (30)$$

where $\mathcal{V}_{1,y}(t)$ is obtained from $\mathcal{V}_1(t)$ by replacing $\Theta(s)$ with $y^T(s)y(s)$. Then, it follows from (29) and (30) that:

$$\int_0^t e^{-2\alpha_1 t} \mathbf{E}\{y^T(s)y(s)\}\mathrm{d}s \le \gamma^2 \mu_2 e^c \int_0^t e^{-\lambda(t-s)} \bar{w}^T(s)\bar{w}(s)\mathrm{d}s. \quad (31)$$

The integration of both sides of (31) from $t = 0$ to ∞ yields

$$\int_0^\infty e^{-2\alpha_1\, s} \mathbf{E}\{y^T(s)y(s)\}\mathrm{d}s \le \tilde{\gamma}^2 \int_0^\infty \bar{w}^T(s)\bar{w}(s)\mathrm{d}s. \quad (32)$$

Case 2: If $t \in \mathcal{H}_{2,n}$, by similar procedures as in Case 1, it can be derived that

$$\mathbf{E}\{V(t)\} \le e^{-2\alpha_1 t + r(t,0)} V_1(0)/\mu_2 - \mathcal{V}_2(t). \quad (33)$$

where

$$\mathcal{V}_2(t) = \frac{1}{\mu_2} \sum_{k=0}^{n} \int_{d_k}^{d_k+s_k} e^{-2\alpha_1(t-s)+r(t,s)} \mathbf{E}\{\Theta(s)\}\mathrm{d}s + \sum_{k=1}^{n} \int_{d_{k-1}+s_{k-1}}^{d_k} e^{-2\alpha_1(t-s)+r(t,s)} \mathbf{E}\{\Theta(s)\}\mathrm{d}s + \int_{d_n+s_n}^{t} e^{-2\alpha_1(t-s)+r(t,s)} \mathbf{E}\{\Theta(s)\}\mathrm{d}s.$$

By similar procedure used in Case 1, we can obtain (32). Therefore, it can be concluded that the system (11) possesses a weighted L_2-gain level $\tilde{\gamma}$.

Next, the exponential stability of the system (11) will be proven in a disturbance-free situation based on the above discussion. For Case 1 ($t \in \mathcal{H}_{1,n}$), using (27) and Assumptions 1 and 2 yields

$$\mathbf{E}\{V(t)\} \le e^c e^{-\lambda t} V_1(0). \quad (34)$$

For Case 2 ($t \in \mathcal{H}_{2,n}$), applying (33) and Assumptions 1 and 2, we have

$$\mathbf{E}\{V(t)\} \le \frac{1}{\mu_2} e^c e^{-\lambda t} V_1(0). \quad (35)$$

Let $d_1 = \min_{j\in\{1,2\}}\{\lambda_{\min}(P_j)\}$, $d_2 = \max_{j\in\{1,2\}}\{\lambda_{\max}(P_j)\}$, and $d_3 = d_2 + h\lambda_{\max}(Q_1) + \frac{h^2}{2}\lambda_{\max}(R_1 + Z_1)$. By the definition of functional $V_j(t)$ in (19), we know that there exists a scalar $\Theta > 1$ such that

$$\mathbf{E}\{V(t)\} \ge d_1 \mathbf{E}\{||\xi(t)||^2\},\ V_1(0) \le \Theta d_3 ||\varphi||_h^2. \quad (36)$$

Combining (34), (35), and (36) yields

$$\mathbf{E}\{||\xi(t)||^2\} \le \frac{\Theta d_3}{d_1} e^{-\lambda t} ||\varphi||_h^2.$$

Therefore, system (11) with $\bar{w}(t) = 0_{(p+v)\times 1}$ is mean-square exponentially stable, which completes the proof. ■

Note that Theorem 1 provides sufficient conditions for system (11) to realize the mean-square exponential stability with a weighted L_2-gain performance level. Based on Theorem 1, we can conclude the following result to implement the joint-design of observer gain L in (3), weighting matrix Ψ in (6), and control gain K in (7).

Theorem 2: Assume that $\gamma > 0$, $\chi > 0$, $\epsilon_j \ge 0$, $\rho_j > 0$ $(j = 1, 2)$ are known. For scalars $\sigma_1 \in [0, 1)$, $T_{D1} > 0$, $T_{D2} > 1$, $\alpha_j > 0$, $\mu_j > 1$ $(j = 1, 2)$ and $h > 0$ satisfying (14), if there exist matrices $\Psi > 0$, $P_j > 0$, $Q_j > 0$, $R_j > 0$, $Z_j > 0$, $J > 0$, S_j, M_j, N_j $(j = 1, 2)$, $\tilde{L}$, W and U with appropriate dimensions such that (15), (17), and the following LMIs hold:

$$\begin{bmatrix} \tilde{\Xi}_1 + \mathcal{E}_2^T J \mathcal{E}_2 & \mathcal{E}_1^T U^T B^T B & 0_{(15n+p+v)\times n} \\ * & \mathrm{He}\{-B^T B W\} & B^T P_{11}^T - W^T B^T \\ * & * & -J \end{bmatrix} < 0 \quad (37)$$

$$\tilde{\Xi}_2 = \begin{bmatrix} \tilde{\Xi}_{211} & \tilde{\Xi}_{212} \\ * & \tilde{\Xi}_{222} \end{bmatrix} < 0 \quad (38)$$

where $\mathcal{E}_1 = [0_{n\times 2n}\ I_n\ -I_n\ 0_{n\times 2n}\ I_n\ 0_{n\times(8n+p+v)}]$, $\mathcal{E}_2^T = [I_n\ 0_{n\times(12n+p+v)}\ I_n\ 0_{n\times n}]$, $P_1 = \mathrm{diag}\{P_{11}, P_{12}\}$, $P_2 = \mathrm{diag}\{P_{21}, P_{12}\}$, $\tilde{\Xi}_1$ and $\tilde{\Xi}_2$ are obtained from Ξ_1 and Ξ_2 by replacing $P_{11}BK$, $P_{12}LC$, $-P_j(R_j + Z_j)^{-1}P_j$ with BU, $\tilde{L}C$, $-2\epsilon_j P_j + \epsilon_j^2(R_j + Z_j)$, then system (11) is mean-square exponentially stable with a weighted L_2-gain. Furthermore, observer gain matrix L and control gain matrix K are obtained as $L = P_{12}^{-1}\tilde{L}$ and $K = W^{-1}U$.

Proof: By Schur complement and Lemma 2 in [23], condition (37) can be rewritten as

$$\begin{bmatrix} \tilde{\Xi}_1 + \mathcal{E}_2^T J \mathcal{E}_2 & \mathcal{E}_1^T U^T B^T B \\ * & \bar{\Lambda}_{33} \end{bmatrix} < 0 \quad (39)$$

where $\bar{\Lambda}_{33} = \mathrm{He}\{-B^T B W\} + \Lambda_{33}$, $\Lambda_{33} = (P_{11}B - BW)^T J^{-1} (P_{11}B - BW)$. Applying Lemma 2 in [23], one has

$$\begin{bmatrix} \tilde{\Xi}_1 + \mathcal{E}_2^T J \mathcal{E}_2 & \mathcal{E}_1^T (W^{-1}U)^T \\ * & -\Lambda_{33}^{-1} \end{bmatrix} < 0.$$

Authorized licensed use limited to: Cornell University Library. Downloaded on May 23,2021 at 20:27:28 UTC from IEEE Xplore. Restrictions apply.

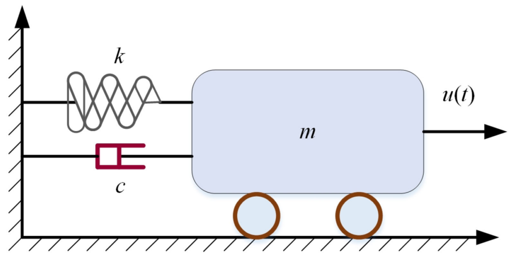


Figure 3. Schematic diagram of the mass-spring-damper mechanical system.

Using Schur complement, we have

$$\tilde{\Xi}_1 + \mathcal{E}_2^T J \mathcal{E}_2 + \mathcal{E}_1^T (W^{-1}U)^T (P_{11}B - BW)^T J^{-1} \times (P_{11}B - BW) W^{-1} U \mathcal{E}_1 < 0$$

which yields

$$\tilde{\Xi}_1 + \mathrm{He}\{\mathcal{E}_2 (P_{11}B - BW) W^{-1} U \mathcal{E}_1\} < 0. \quad (40)$$

Since $((R_1+Z_1)^{-\frac{1}{2}}P_1 - v_1(R_1+Z_1)^{\frac{1}{2}})^T((R_1+Z_1)^{-\frac{1}{2}}P_1 - v_1(R_1+Z_1)^{\frac{1}{2}}) \geq 0$, it is clear that

$$-P_1(R_1+Z_1)^{-1}P_1 \leq -2v_1P_1 + v_1^2(R_1+Z_1). \quad (41)$$

Define $\tilde{L} = P_{12}L$ and $U = WK$. Based on (40) and (41), condition (18) with $j=1$ can be guaranteed. Similarly, based on (38) and (41), condition (18) with $j=2$ holds. This ends the proof. ■

Remark 3: The optimal performance level $\gamma_{\min}$ can be obtained by solving the following minimization problem:

$$\gamma_{\min} = \min\{\gamma | \gamma \text{ satisfying LMIs (37) and (38)}\}. \quad (42)$$

Remark 4: It should be mentioned that Theorem 2 gives an LMI-based solution to co-design the observer, the event trigger, and the controller for the stochastic NCS under aperiodic DoS attacks. Note that the nonlinear terms in Theorem 1 cannot be dealt with effectively by the congruent transformation presented in most existing literature. In this article, a new method is introduced to eliminate the coupling terms in Theorem 1. As a result, Theorem 2 is deduced and gain matrices L, Ψ, and K can be calculated simultaneously. The proposed decoupled method can also be applied to handle the nonlinear matrix inequality in [22, Theor. 7] when gain matrices are not known in advance.

IV. An Illustrative Example

In this section, a mass-spring-damper mechanical system (see Fig. 3) is used to illustrate the effectiveness and applicability of the proposed approaches. According to Newton's second law, the dynamic equation is governed by

$$m\ddot{z}(t) + c\dot{z}(t) + kz(t) = u(t)$$

where m, $z(t)$, c, k, and $u(t)$ is the mass, the displacement, the damping coefficient, the spring stiffness and the external force, respectively. By introducing $x_1(t) = z(t)$ and $x_2(t) = \dot{z}(t)$, one has

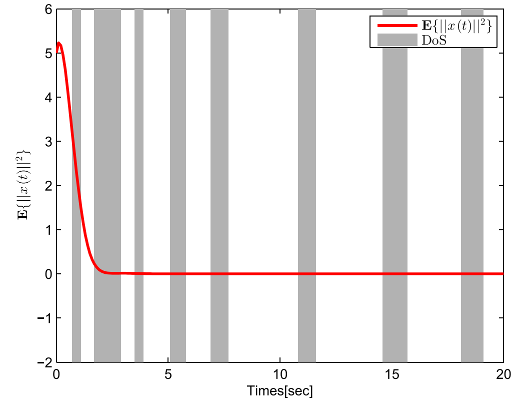


Figure 4. Trajectory of $\mathbf{E}\{||x(t)||^2\}$.

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = -\frac{k}{m}x_1(t) - \frac{c}{m}x_2(t) + \frac{1}{m}u(t). \end{cases} \quad (43)$$

Let $x(t) = [x_1(t)\ x_2(t)]^T$, $m=2$, $k=c=4$, system (43) can be transformed into (1) with

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, B_w = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$$

$$D = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.6 \end{bmatrix}, C = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T, x(0) = \begin{bmatrix} 2 \\ -1 \end{bmatrix}.$$

The $\varpi(t)$ in (1) is taken as a zero-mean Wiener process with $\psi(x(t)) = 1.5||x(t)||\sin(x(t))$. Then, it can be confirmed that $\chi = 2.25$. Choose $T_{D1} = 2$, $T_{D2} = 2$, $\alpha_1 = 0.2, \alpha_2 = 0.1, \mu_1 = \mu_2 = 1.02$ and $h = 0.1\ s$. According to (14), we know that $\lambda = 0.0502$. By Assumptions 1 and 2, it can be obtained that the allowable maximum values of attack frequency and duration are $\frac{20}{T_{D1}} = 10\ Hz$ and $\frac{20}{T_{D2}} = 10\ s$. We assume that attack frequency $n(t,0) = 8$ and attack duration $|\Pi(t,0)| = 8\ s$. The corresponding aperiodic DoS attacks are depicted in Fig. 4 (or Figs. 6–8). Set $\sigma_0 = 0.35$, $\sigma_1 = 0.4$, $v_1 = v_2 = 8$, $\rho_1 = 30$, $\rho_2 = 40$ and $\gamma = 1$. By solving LMIs (37) and (38) in Theorem 2, matrices Ψ, K and L are obtained as

$$\Psi = \begin{bmatrix} 0.5990 & 0.9742 \\ 0.9742 & 5.3704 \end{bmatrix}, \quad K = \begin{bmatrix} -0.0116 & -0.1278 \end{bmatrix}$$

$$L = \begin{bmatrix} 2.4990 & -2.8125 \end{bmatrix}^T.$$

The evolution of $\mathbf{E}\{||x(t)||^2\}$, the relationship between the releasing instants and the inner-event intervals under event-triggered scheme (6) with $\kappa_1 = 0.1$ and $\kappa_2 = 1$, system state $x(t)$, observer state $\hat{x}(t)$ and control input $u(t)$ are shown in Figs. 4–8, respectively. From Figs. 4 and 6, it can be seen that the mass-spring-damper mechanical system is stabilized by the designed controller. Results in Fig. 5 indicate that the proposed event-triggered scheme can eliminate the adverse effects of aperiodic DoS attacks while reducing the amount of transmitted data. Compared with the result in [12], the number of transmitted data packets generated by our proposed event-triggered scheme (6) is declined by 7.

Authorized licensed use limited to: Cornell University Library. Downloaded on May 23,2021 at 20:27:28 UTC from IEEE Xplore. Restrictions apply.

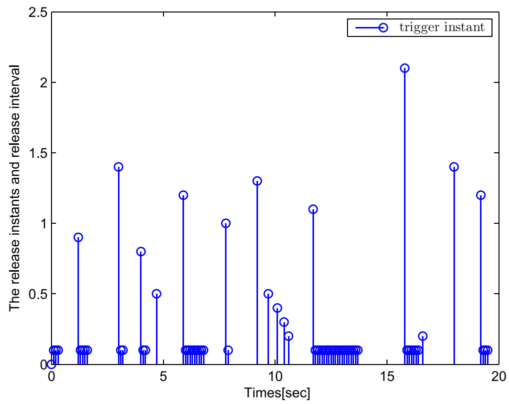


Figure 5. Releasing instants and intervals.

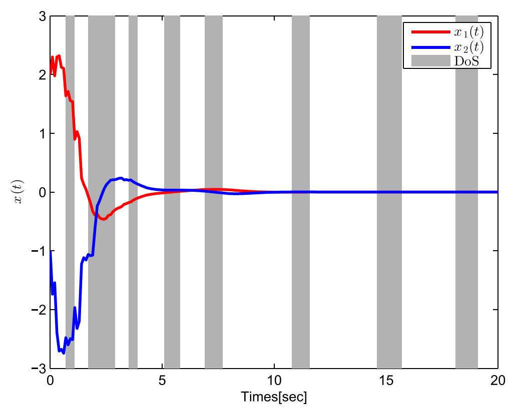


Figure 6. State responses $x(t)$.

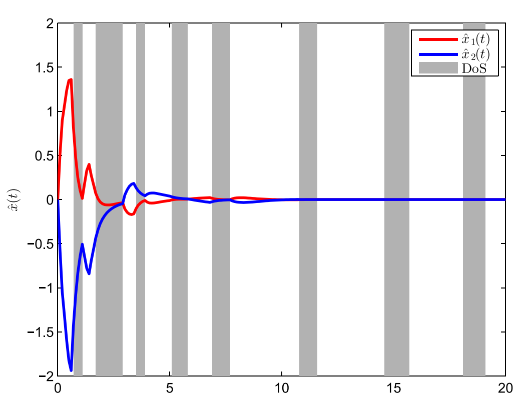


Figure 7. State estimations $\hat{x}(t)$.

Next, we will evaluate the weighted L_2-gain performance of the closed-loop control system. The external disturbance is given as

$$w(t) = \begin{cases} \cos(t), & t \in [0, 8], \\ 0, & \text{else}. \end{cases}$$

By a simple calculation, we achieve that

$$\sqrt{\frac{\int_0^\infty e^{-2\alpha_1 s} y^T(s) y(s) \mathrm{d}s}{\int_0^\infty \bar{w}^T(s) \bar{w}(s) \mathrm{d}s}} \approx 0.6938 \leq \tilde{\gamma} = 2.8229 \quad (44)$$

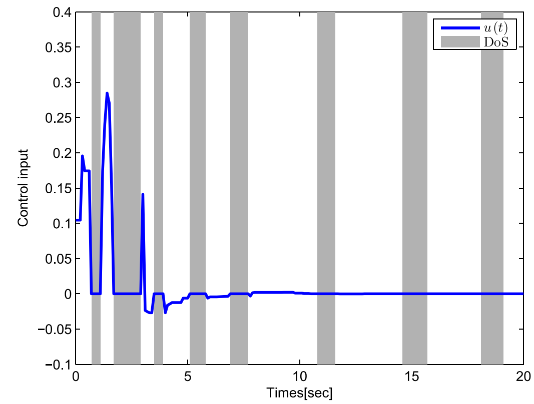


Figure 8. Control input $u(t)$.

TABLE I
OBTAINED $\gamma_{\min}$ WITH DIFFERENT VALUES OF σ_1

σ_1	0.1	0.25	0.4	0.55
$\gamma_{\min}$	0.8475	0.8558	0.8581	0.8584

TABLE II
OBTAINED $\gamma_{\min}$ WITH DIFFERENT VALUES OF h

h	0.02	0.05	0.1	0.12
$\gamma_{\min}$	0.7667	0.7966	0.8581	0.8892

which indicates that conditions in Theorem 2 ensure the required degree of disturbance attenuation and further demonstrates that our proposed approaches are effective.

By setting different event-triggered threshold σ_1, the minimum L_2-gain performance level can be estimated by solving optimization problem (42). The relation between σ_1 and $\gamma_{\min}$ is revealed in Table I , from which we can observe that $\gamma_{\min}$ becomes smaller with the decreasing of σ_1. Moreover, by Theorem 2 and Remark 3, one can discover the link between the sampling period h and the admissible smallest value of γ. As can be seen from Table II, a smaller h will lead to a better system performance.

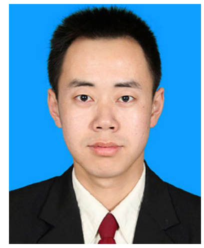
V. CONCLUSION

This article has addressed the resilient controller design problem for stochastic NCSs by considering aperiodic DoS jamming attacks at communication channels. An observer-based adaptive event generator was proposed and adopted to reduce the utilization of communication resources while alleviating the impact of aperiodic DoS attacks. A secure controller was designed and implemented to guarantee the resulting switched system to be mean-square exponentially stable with a desired L_2-gain performance level. A co-design criterion of the observer, the controller, and the event-triggered parameters were provided. The availability and advantages of the new design techniques in a mass-spring-damper mechanical system were verified. Based on our current work, further investigation will focus on distributed NCSs and networked interconnected systems.

Authorized licensed use limited to: Cornell University Library. Downloaded on May 23,2021 at 20:27:28 UTC from IEEE Xplore. Restrictions apply.

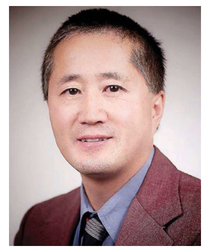
References

[1] R. J. Anderson and M. W. Spong, "Bilateral control of teleoperators with time delay," *IEEE Trans. Autom. Control*, vol. 34, no. 5, pp. 494–501, May 1989.
[2] P. Antsaklis and J. Baillieul, "Special issue on technology of networked control systems," *Proc. IEEE*, vol. 95, no. 1, pp. 5–8, 2007.
[3] P. Papadimitratos, A. D. L. Fortelle, K. Evenssen, R. Brignolo, and S. Cosenza, "Vehicular communication systems: Enabling technologies, applications, and future outlook on intelligent transportation," *IEEE Commun. Mag.*, vol. 47, no. 11, pp. 84–95, Nov. 2009.
[4] X. Lv, J. Fei, and Z. Feng, "State feedback control for uncertain networked control systems with disturbances," *Int. J. Innov. Comput. I*, vol. 13, no. 3, pp. 751–765, 2017.
[5] M. Gao, S. Jiang, and F. Pan, "Fault detection for time-delayed networked control systems with sensor saturation and randomly occurring faults," *Int. J. Innov. Comput. I*, vol. 14, no. 4, pp. 1511–1525, 2018.
[6] Y. Wang, S. Ding, D. Xu, and B. Shen, "An H_∞ fault estimation scheme of wireless networked control systems for industrial real-time applications," *IEEE Trans. Control Syst. Technol.*, vol. 22, no. 6, pp. 2073–2086, Nov. 2014.
[7] L. Cheng, J. Wu, H. Gao, and T. Han, "Optimal performance of networked control systems with packet dropouts and bandwidth constraints," *ICIC Express Lett.*, vol. 11, no. 4, pp. 863–870, 2017.
[8] Q. Liu, "State estimation for networked control systems at the extreme data-rate limit," *ICIC Express Lett. Part B: Appl.*, vol. 8, no. 1, pp. 117–122, 2017.
[9] H. Shisheh Foroush and S. Martínez, "On triggering control of single-input linear systems under pulse-width modulated DoS signals," *SIAM J. Control Optim.*, vol. 54, no. 6, pp. 3084–3105, 2016.
[10] S. Feng and P. Tesi, "Resilient control under denial-of-service: Robust design," *Automatica*, vol. 79, pp. 42–51, 2016.
[11] C. De Persis and P. Tesi, "Input-to-state stabilizing control under denial-of-service," *IEEE Trans. Autom. Control*, vol. 60, no. 11, pp. 2930–2944, Nov. 2015.
[12] S. Hu, D. Yue, X. Xie, X. Chen, and X. Yin, "Resilient event-triggered controller synthesis of networked control systems under periodic DoS jamming attacks," *IEEE Trans. Cybern.*, vol. 49, no. 12, pp. 4271–4281, Dec. 2019.
[13] J. Liu, Z. Wu, D. Yue, and J. H. Park, "Stabilization of networked control systems with hybrid-driven mechanism and probabilistic cyber attacks," *IEEE Trans. Syst. Man Cybern. Syst.*, early access, Jan. 4, 2019, doi: 10.1109/TSMC.2018.2888633.
[14] H. Song, P. Shi, W. Zhang, C. C. Lim, and L. Yu, "Distributed H_∞ estimation in sensor networks with two-channel stochastic attacks," *IEEE Trans. Cybern.*, vol. 50, no. 2, pp. 465–475, Feb. 2020.
[15] Y. Yuan, H. Yuan, L. Guo, H. Yang, and S. Sun, "Resilient control of networked control system under DoS attacks: A unified game approach," *IEEE Trans. Ind. Inf.*, vol. 12, no. 5, pp. 1786–1794, Oct. 2016.
[16] Y. Yuan, H. Yuan, D. W. C. Ho, and L. Guo, "Resilient control of wireless networked control system under denial-of-service attacks: A cross-layer design approach," *IEEE Trans. Cybern.*, vol. 50, no. 1, pp. 48–60, Jan. 2020.
[17] J. Qin, M. Li, L. Shi, and X. Yu, "Optimal denial-of-service attack scheduling with energy constraint over packet-dropping networks," *IEEE Trans. Autom. Control*, vol. 63, no. 6, pp. 1648–1663, Jun. 2018.
[18] X. Chen, Y. Wang, and S. Hu, "Event-based robust stabilization of uncertain networked control systems under quantization and denial-of-service attacks," *Inf. Sci.*, vol. 459, pp. 369–386, 2018.
[19] X. Su, Y. Wen, P. Shi, and H. K. Lam, "Event-triggered fuzzy filtering for nonlinear dynamic systems via reduced-order approach," *IEEE Trans. Fuzzy Syst.*, vol. 27, no. 6, pp. 1215–1225, Jun. 2019.
[20] P. Shi, H. Wang, and C. C. Lim, "Network-based event-triggered control for singular systems with quantizations," *IEEE Trans. Ind. Electron.*, vol. 63, no. 2, pp. 1230–1238, Feb. 2016.
[21] W. Xing, P. Shi, R. K. Agarwal, and L. Li, "Robust H_∞ pinning synchronization for complex networks with event-triggered communication scheme," *IEEE Trans. Circuits Syst. I: Reg. Paper*, early access, Jul. 27, 2020, doi: 10.1109/TCSI.2020.3004170.
[22] L. Wu, Y. Gao, J. Liu, and H. Li, "Event-triggered sliding mode control of stochastic systems via output feedback," *Automatica*, vol. 82, pp. 79–92, 2017.
[23] X. Chang and G. Yang, "New results on output feedback H_∞ control for linear discrete-time systems," *IEEE Trans. Autom. Control*, vol. 59, no. 5, pp. 1355–1359, May 2014.



Ning Zhao is currently pursing the Ph.D. degree in control science and engineering with the College of Intelligent Systems Science and Engineering, Harbin Engineering University, Harbin, China.

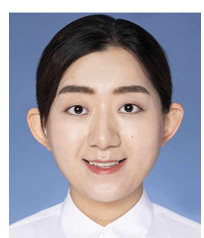
His current research interests include networked control systems and event-triggered control.



Peng Shi (Fellow, IEEE) received the Ph.D. degree in electrical engineering from the University of Newcastle, Callaghan, Australia, in 1994.

He is currently a Professor with the University of Adelaide, Adelaide, Australia. His research interests include system and control theory, intelligent systems, and operational research. He has served on the editorial board of a number of journals, including *Automatica*; IEEE Transactions on (Automatic Control; Cybernetics; Fuzzy Systems; Circuits and Systems); IEEE Access; *IEEE Control Systems Letters*; *Information Sciences and Signal Processing*.

Dr. Shi was awarded the Doctor of Science degree from the University of Glamorgan, Wales in 2006; and the Doctor of Engineering degree from the University of Adelaide, Australia in 2015. He is a Member-at-Large of Board of Governors, IEEE SMC Society, and an IEEE SMCS Distinguished Lecturer. He is a Fellow of the Institution of Engineering and Technology and the Institute of Engineers, Australia.



Wen Xing (Member, IEEE) received the Ph.D. degree in intelligent systems science and engineering from Harbin Engineering University, Harbin, China, in 2020. From 2017 to 2019, she was a Visiting Student with the School of Electrical and Electronic Engineering, University of Adelaide, Adelaide, Australia.

She is currently a Lecturer with Harbin Engineering University. Her research interests include complex networks, multiagent systems, and distributed cooperative control.

Jonathon Chambers (Fellow, IEEE) received the Ph.D. and D.Sc. degrees in signal processing from the Imperial College of Science, Technology and Medicine (Imperial College London), London, U.K., in 1990 and 2014, respectively.

He is Emeritus Professor of Signal and Information Processing and Former Head of the School of Engineering with the University of Leicester, Leicester, U.K. He is also an International Honorary Dean and Guest Professor with Harbin Engineering University, China. His research interests include adaptive signal processing and machine learning and their applications.

Dr. Chambers is a Fellow of the Royal Academy of Engineering, U.K., and the Institution of Electrical Engineers. He has served as an Associate Editor for the IEEE Transactions on Signal Processing over the periods 1997-1999, 2004-2007, and as a Senior Area Editor between 2011-2014. Since 2016 he has been an Area Editor for the *Elsevier DSP* journal.

Authorized licensed use limited to: Cornell University Library. Downloaded on May 23,2021 at 20:27:28 UTC from IEEE Xplore. Restrictions apply.